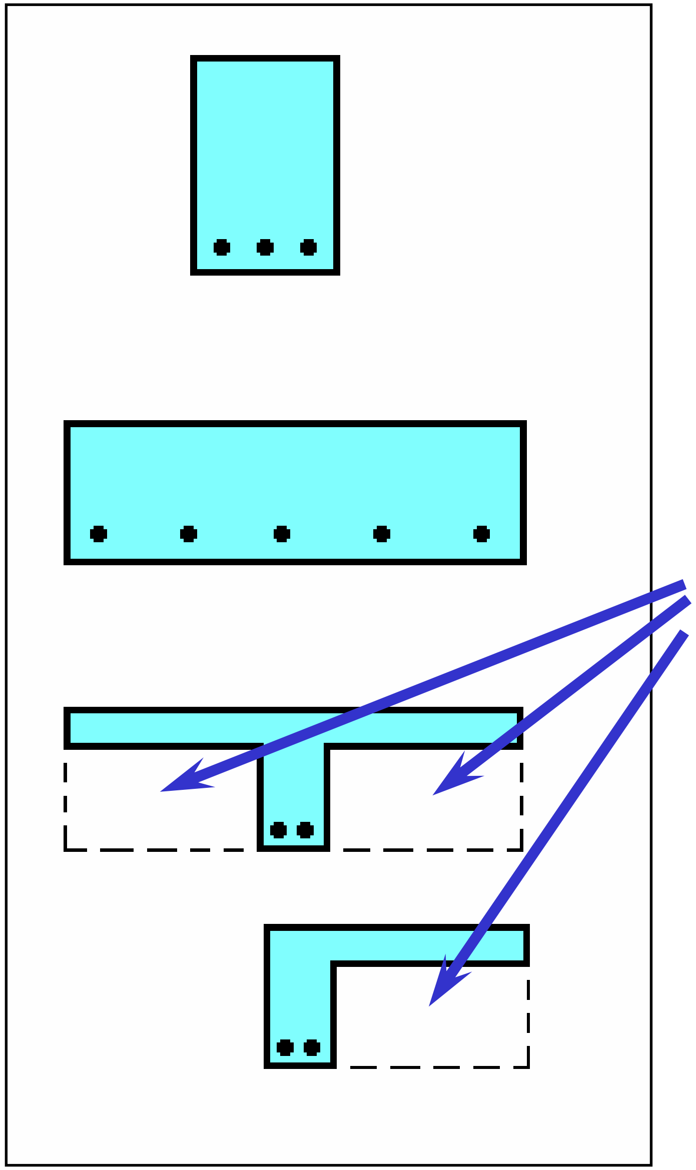
ENDP3112 Structural Concrete Design

16. Beam-and-Slab Design

- Beam-and-Slab System
- How does the slab work?
- L- beams and T- beams
- Holding beam and slab together

BEAM - AND - SLAB SYSTEM



Here is a conventional beam:

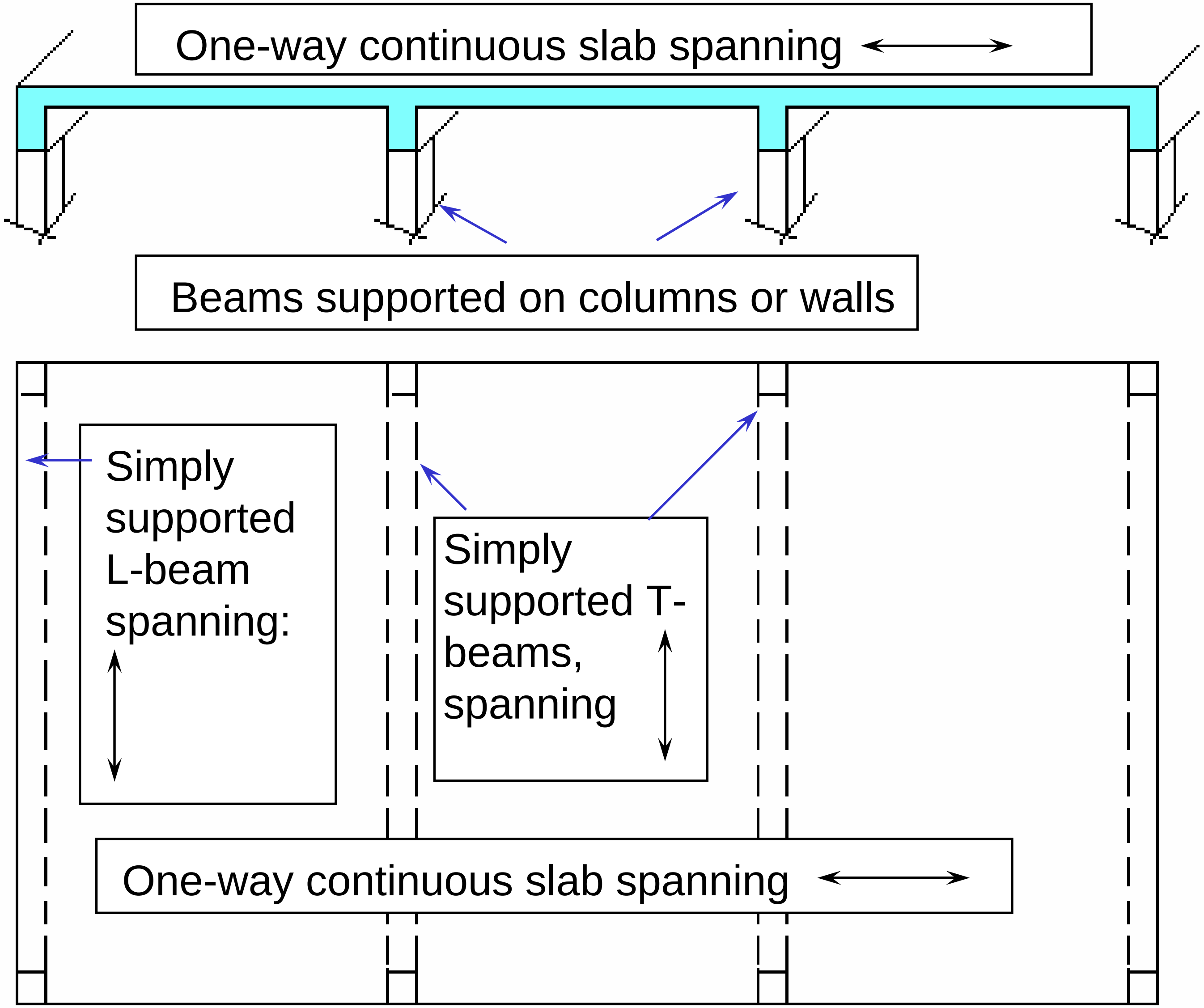
Here is a wide beam:

Here is a T-beam:

and an L-beam:

We can save all this dead weight - ***provided that*** the reduced web can resist the induced shear actions.

So we can design a floor system, using beam and slab design as follows . . .



One-way continuous slab spanning ↔

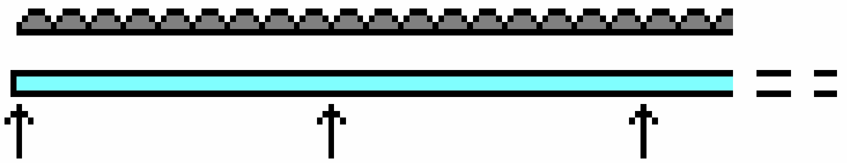
Beams supported on columns or walls

Simply supported L-beam spanning: ↕

Simply supported T-beams, spanning ↕

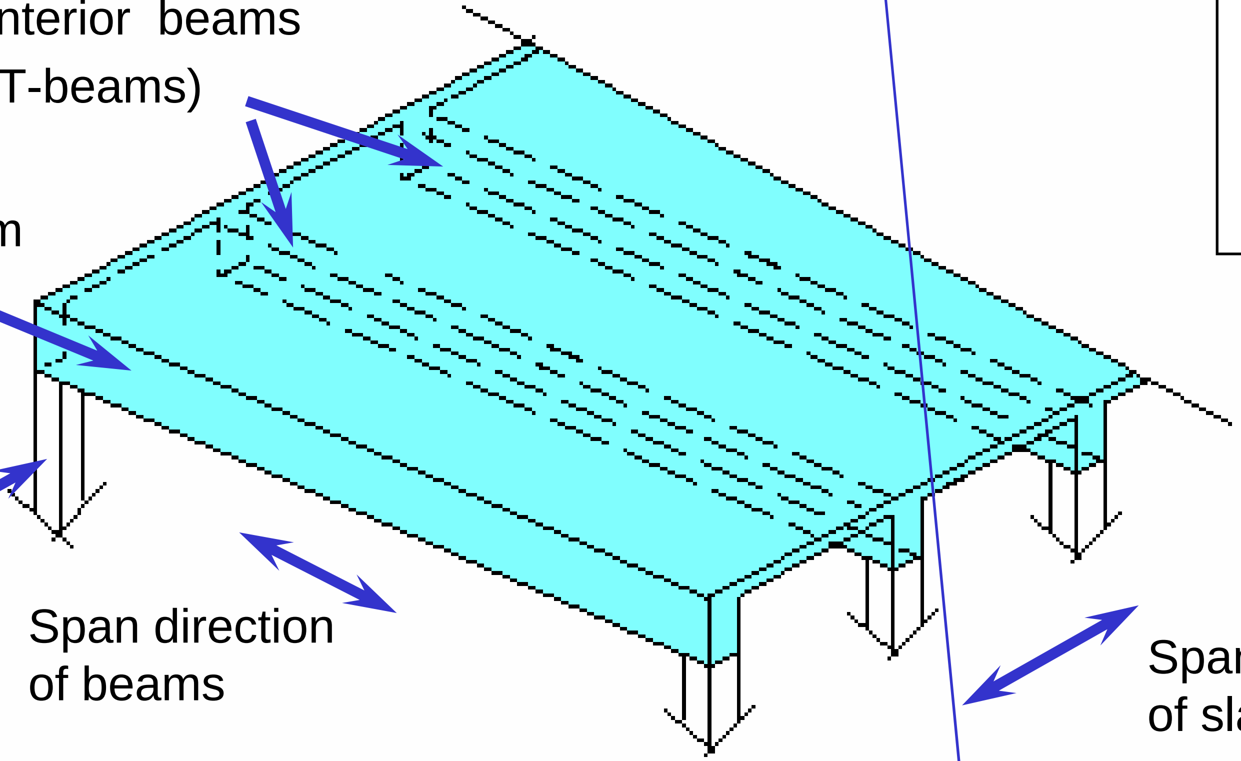
One-way continuous slab spanning ↔

Model for slab design:



These reactions per metre . . .

First, consider the slab . . .



Interior beams

(T-beams)

Edge beam

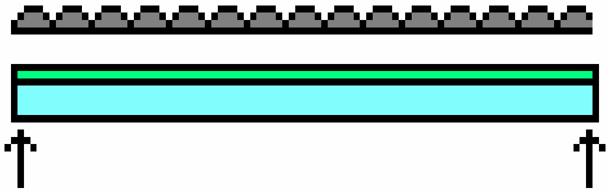
(L-beam)

Support columns

Span direction of beams

Span direction of slab

Model for beam design:



. . . provide this UDL on the beams.

HOW DOES THE SLAB WORK ?

- **Like a one-way continuous slab, supported by walls, which in this case are in fact, beams,**
- **Could proceed to use our methods for continuous beams (slabs in this case) adopting linear-elastic methods (moment distribution, stiffness methods, etc.), or using moment re-distribution.**

However, in most cases a simplified 'coefficient' method is easier. The method applies when :

Cl. 7.2

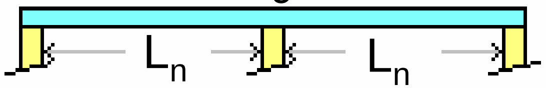
- ***ratio of adjacent span lengths <= 1.2***
- ***loads are essentially UDL - g and q***
- ***q <= 2g***
- ***slab is of uniform section***
- ***rebar layout complies with Code arrangement***
- ***actions at supports are solely due to slab loads***

Let's consider this method . . .

Design moments:

TWO SPAN

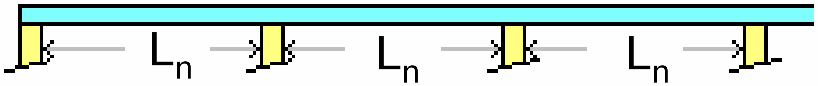
$-\frac{F_d L_n^2}{9}$



$+\frac{F_d L_n^2}{11}$ $+\frac{F_d L_n^2}{11}$

MULTI-SPAN

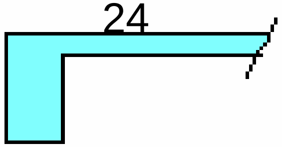
$-\frac{F_d L_n^2}{10}$ $-\frac{F_d L_n^2}{11}$



$+\frac{F_d L_n^2}{11}$ $+\frac{F_d L_n^2}{16}$ $+\frac{F_d L_n^2}{16}$

AT EDGE BEAM

$-\frac{F_d L_n^2}{24}$



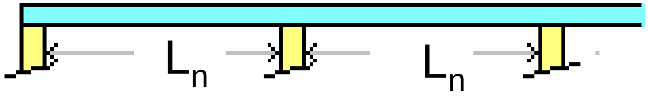
$F_d = 1.2\ g + 1.5\ q$

L_n =clear distance between faces of supports

Design shear forces :

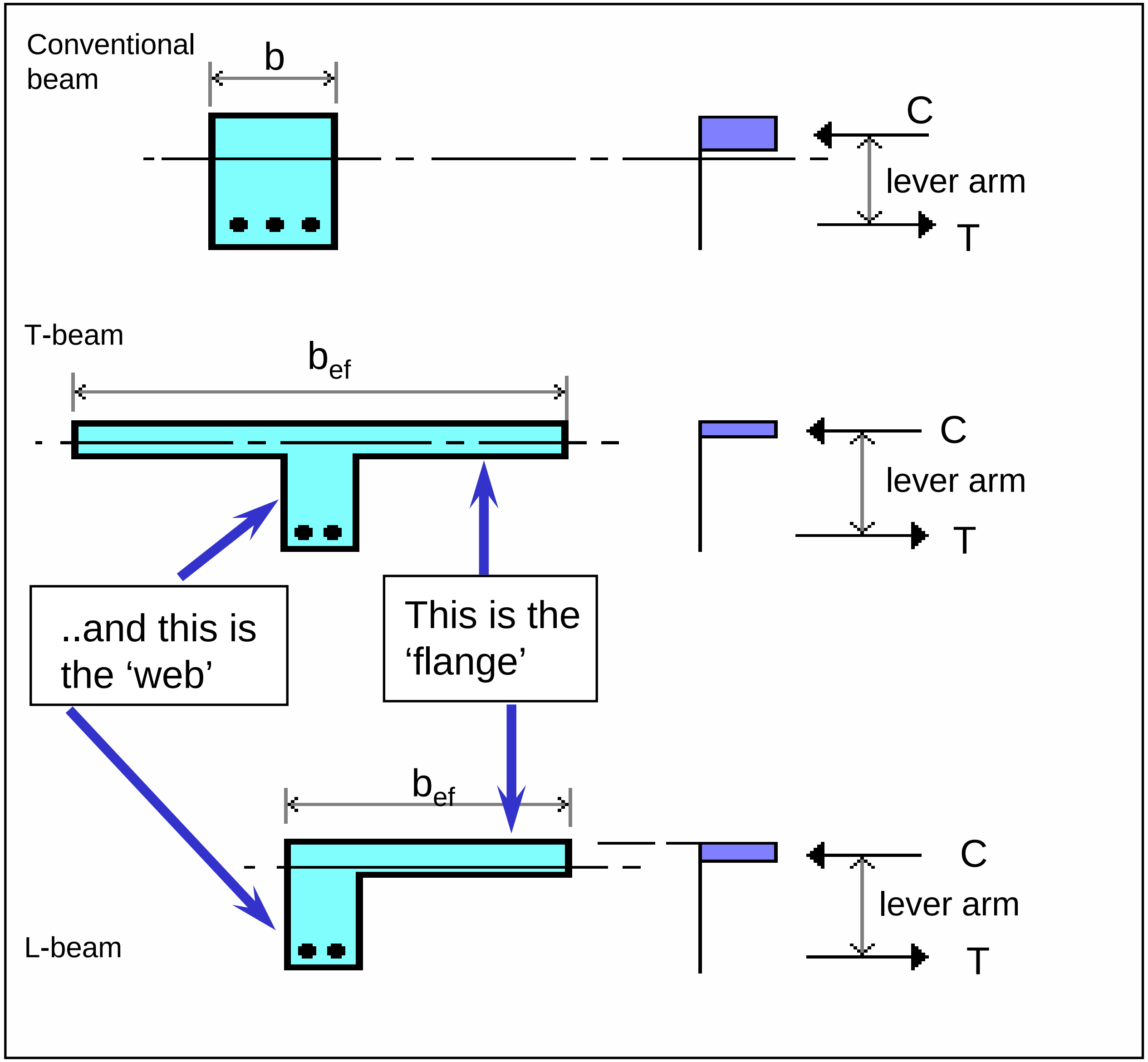
Shear force at RH end: $\frac{1.15\ F_d L_n}{2}$ $\frac{F_d L_n}{2}$

Shear force at LH end: $\frac{F_d L_n}{2}$ $\frac{F_d L_n}{2}$



Shear force at mid span: $\frac{F_d L_n}{7}$ $\frac{F_d L_n}{8}$

T- BEAMS AND L - BEAMS



Since T-beams and L-beams have a greater effective width of compression flange ($b_{ef} > b$), a greater lever arm is available.

So T- and L-beams will have a higher ultimate bending capacity.

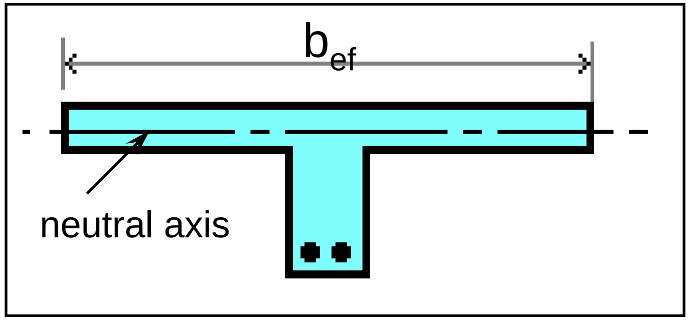
(Likewise, at working moment, the section will be stiffer.)

Simply supported T-beams and L-beams

Bending Resistance

Usually, the neutral axis at ultimate moment is within the flange. So for bending, we proceed as for a rectangular beam, using b_{ef} .

(Sometimes the approximation LA = d - t / 2 is used for preliminary calculations.)



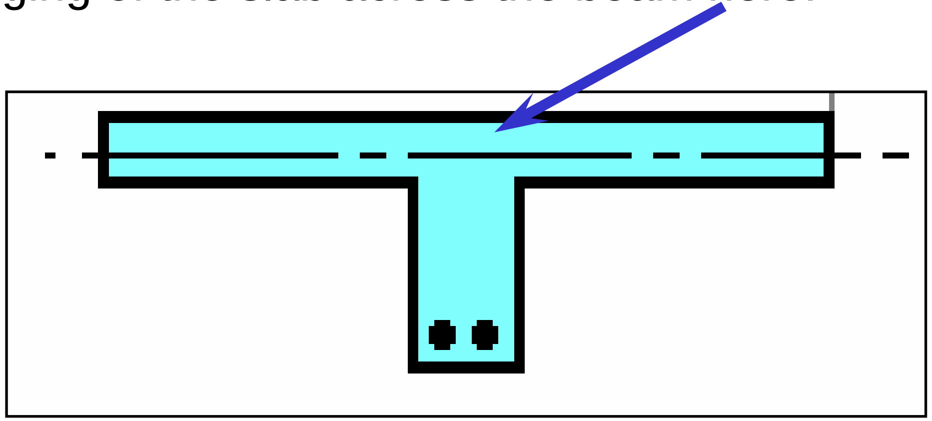
So a T-beam or an L-beam can be designed just as for a rectangular beam, except for the following considerations:

1. Two-dimensional stresses at slab/beam interfaces.
2. An appropriate b_{ef} is selected for the flange.
3. The beam and slab are securely held together.

First, consider (1) . . .

Two dimensional stresses

At mid-span of the beam, the top of the slab is subjected to both compression in the direction of the beam span, and tension caused by the hogging of the slab across the beam here:

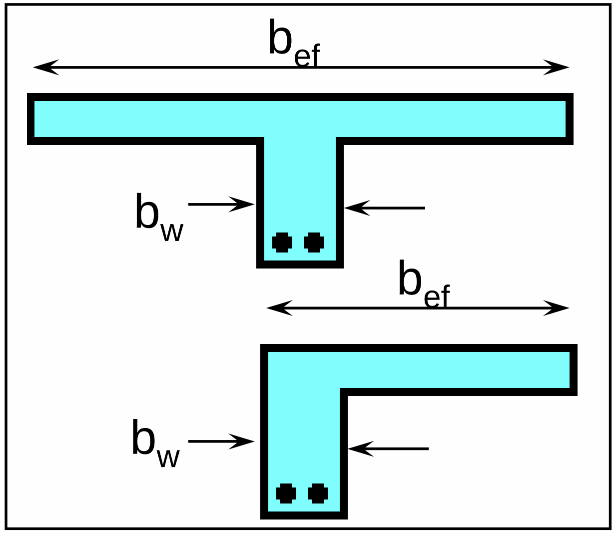


The concrete is subjected to a two-dimensional stress state. At first glance, this may appear to be a problem. However, this is not the case.

The negative moment in the slab is resisted by the tensile rebar. In the beam direction, the ability of the concrete to carry the compressive stress due to positive bending remains relatively unaffected.

Now consider b_{ef} *. . .*

Effective width of beam flange



The width of flange clearly cannot exceed half the distance to the next adjacent beam (otherwise we'd be double-counting).

But the width is also limited by the ability of the section to distribute actions from the web to the flange, and this will be governed by the span of the beam.

AS3600-2001 provides direct guidance:

Cl. 8.8

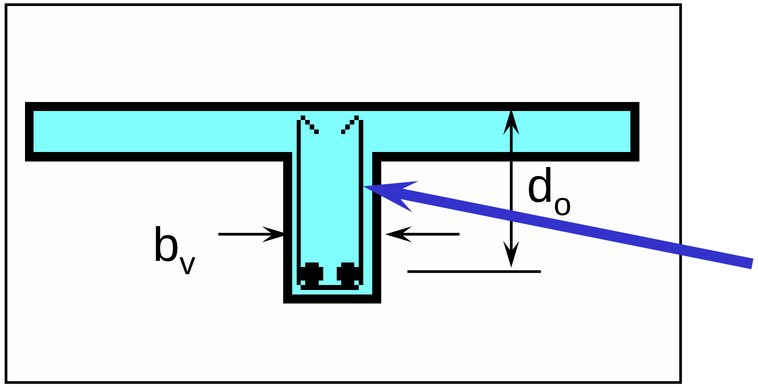
For T-beams $b_{ef} = b_w + 0.2\ a$

For L-beams $b_{ef} = b_w + 0.1\ a$

Where for a simply supported beam,
a = span length L **of the beam**.

So it is important to check b_{ef} before proceeding too far with the design. *Remember not to encroach on the territory of any adjacent beams.*

HOLDING BEAM AND SLAB TOGETHER



Vertical shear action is carried by the web, which includes the common web/flange area.

This is $b_v.d_o$, where b_v and d_o are as shown.

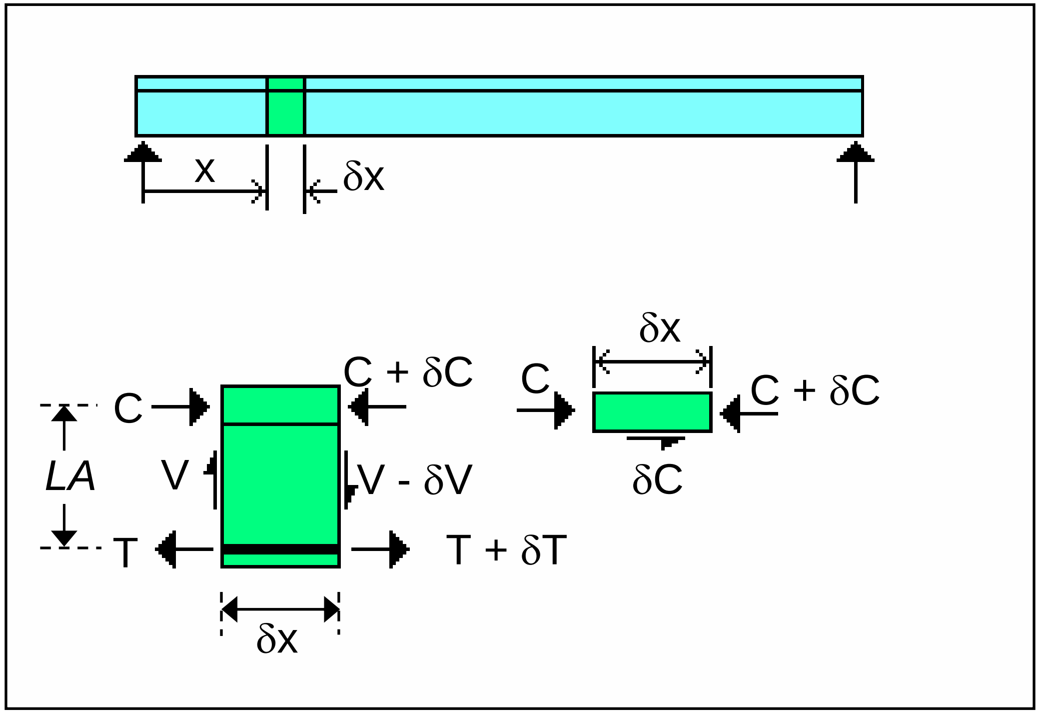
To ensure that the common web/flange is properly engaged in its role of carrying some of the shear force, stirrups are ALWAYS used, and are carried as high as possible.

Note how this explains the use of b_v in all shear resistance formulas considered so far.

Note too that $b_v = b_w$ for reinforced concrete beams. (Footnote: This is not the case for prestressed concrete beams, hence the different terms.)

But there is also a longitudinal (horizontal) shear action to concern us . . .

Resistance to Longitudinal Shear Forces



Over distance δx, a horizontal shear force δC must be provided to ensure integrity of the beam.

From the equilibrium of the element,

δC / δx = V / (Lever Arm)

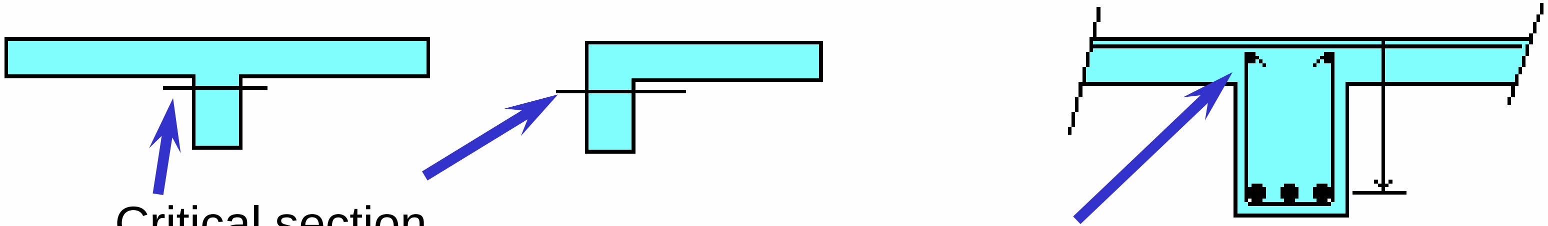
So the rate of change of C is proportional to V, or V* at ultimate load.

So how do we make provision for these stresses? Does the concrete carry the actions, or do we need special rebar arrangements? . . .

SHEAR ON WEB SHEAR PLANE:

δx = Lever Arm

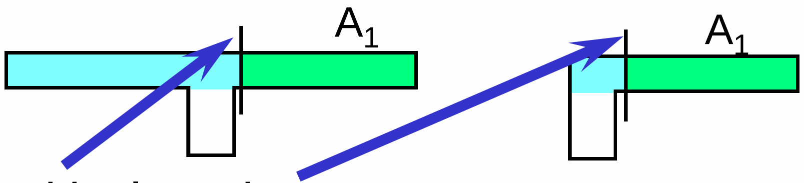
Cl. 8.4



Critical section
Shear force to be resisted = V*

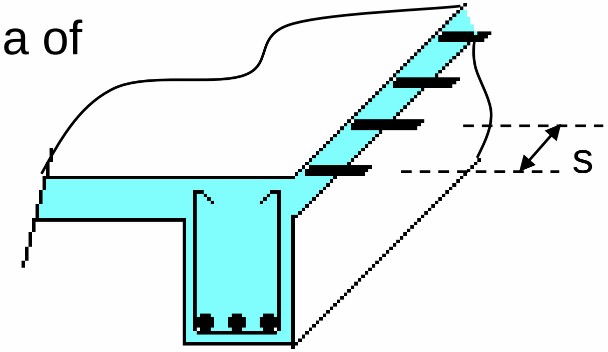
stirrups area A_{sv} at s spacing

SHEAR ON FLANGE SHEAR PLANE:



Critical section
Shear force to be resisted = $V^*.A_1/A_2$

A_2 = total area of flange



Steel contribution

$$V_{uf} = \beta_4 A_s f_{sy} d / s$$

$$+ \beta_5 b_f d f'_{ct}$$

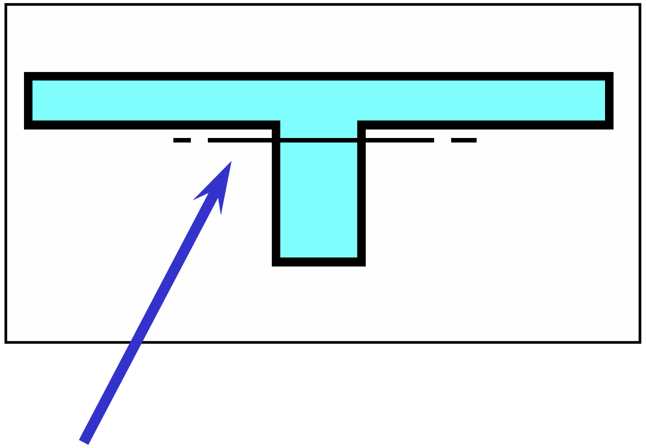
$$< 0.2 f'_c b_f d$$

Concrete contribution

For monolithic construction:
$\beta_4 = 0.9$ and $\beta_5 = 0.5$

Check ϕ Vuf > Longitudinal shear force

Lower Ductility Check



Note that the neutral axis of the *uncracked* section may be below (as shown) or within the flange.

For any concrete beam, we must ensure that, when the first crack occurs, the rebar is adequate to carry the moment which caused the first crack, with an appropriate margin.

This requirement is stated:

$M_{uo} >= (M_{uo})_{min} = 1.2 M_{cr}$

Cl. 8.1.4.1

For T- and L- beams, there is no simple formula for doing this. We must calculate the uncracked second moment of area of the section I_g, the section modulus with respect to the *tension* fibre Z, and proceed from there.

Then $M_{cr} = f'_{cf} \; Z$ and we check that $M_{uo} >= 1.2 \, M_{cr}$

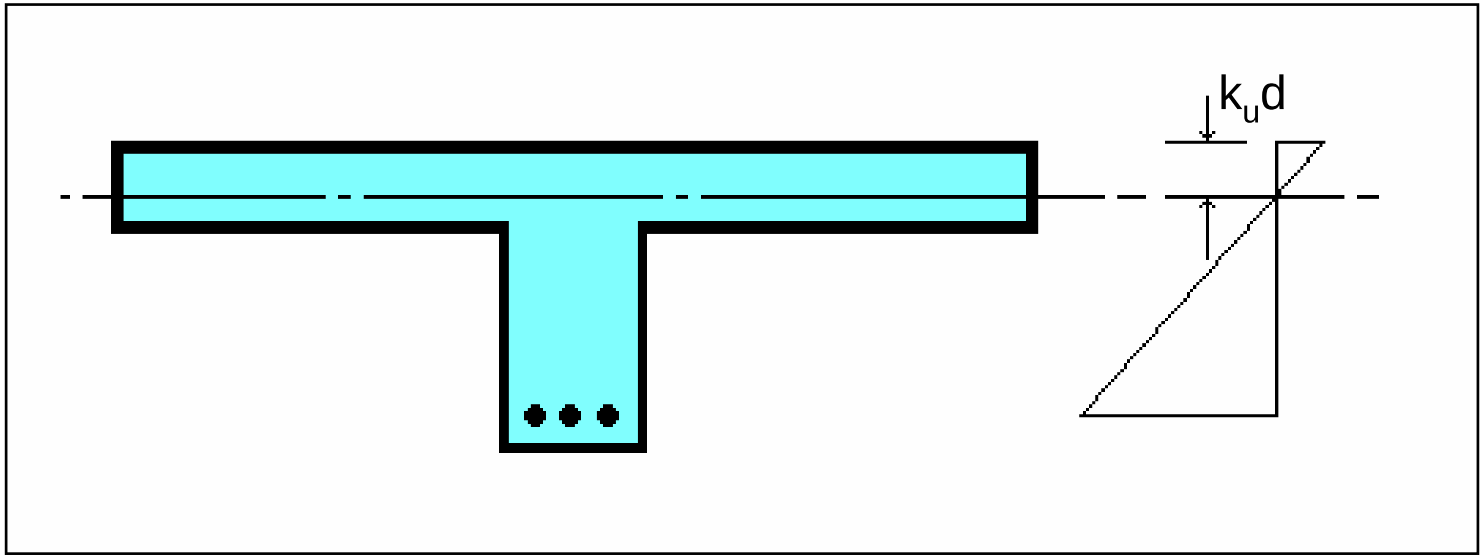
If not, then increase M_{uo} until it is.

Upper Ductility Check

We must also check that the section is not prone to compressive concrete fracture before the rebar has yielded sufficiently to warn the user of a problem.

The check is (as for rectangular beams): $k_u <= 0.4$

This is seldom a problem, since we have a wide compression flange to help us:



But we should *always* check that this is satisfied.

Effective Inertia for Deflection I_{ef}

This can be worked out by Branson's formula, but can be time consuming.

Best to use an approximation to start with, and then use the more complicated method only if deflection appears to be a problem.

A very simple approximation for I_{ef} was provided in AS3600 - 1994:

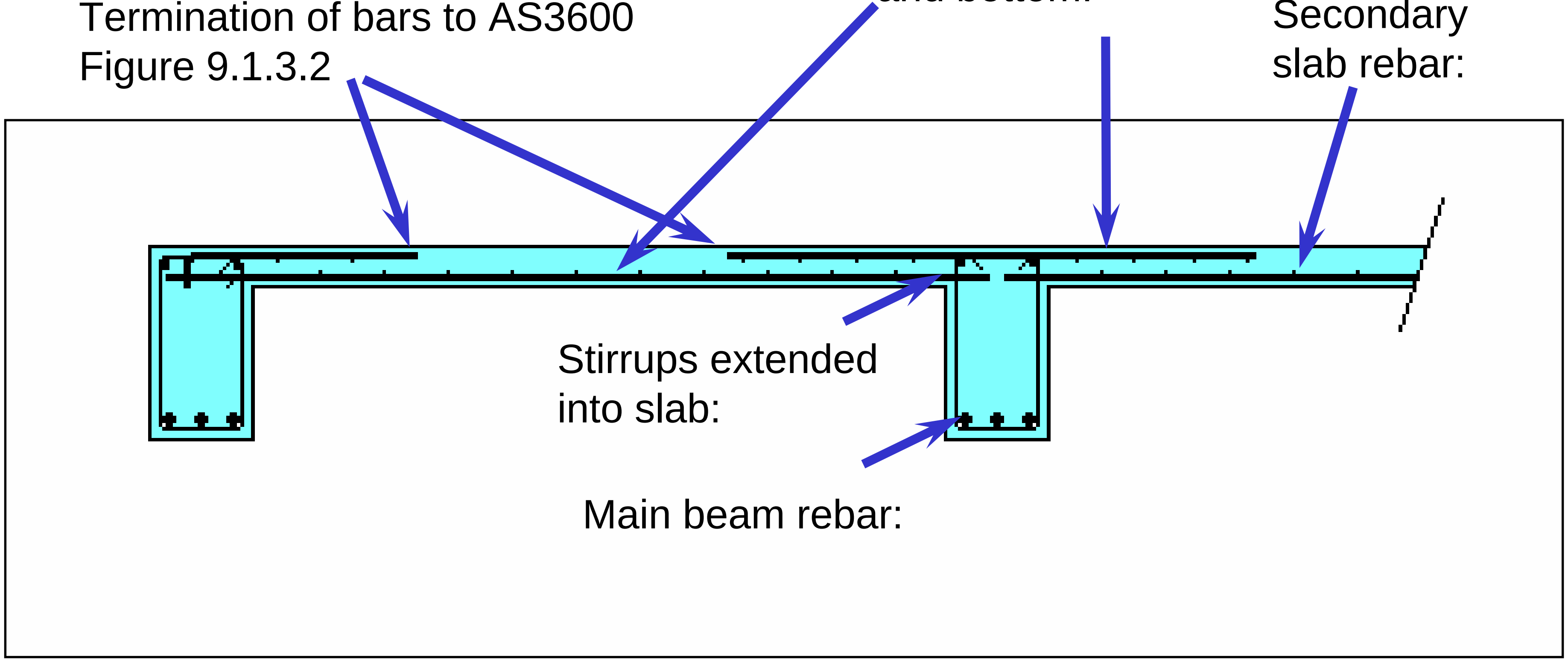
$$I_{ef} = 0.045\, b_{ef}\, d^3 \;\; (0.7 + 0.3\, b_w / b_{ef})^3$$

Using $E_c I_{ef}$, proceed to compute deflections as for a rectangular beam.

If calculated deflection is *much less* than the allowable deflection, then further calculation is not required.

So what about detailing ? . . .

Detailing



Termination of bars to AS3600 Figure 9.1.3.2

Primary slab rebar, top and bottom:

Secondary slab rebar:

Stirrups extended into slab:

Main beam rebar:

The slab is designed as a one-way, continuous slab, spanning across the beam supports.

…and the beam is designed as a simply supported T- (or L-) beam, spanning across its supports.

SUMMARY

- A beam-and-slab system, with a one-way slab, and beams cast compositely with the slab, is a highly efficient floor system.
- The slab is designed as a continuous slab, using theory of continuous beams (slabs), or the simplified ‘coefficient’ method (where applicable i.e. most of the time).
- Edge beams act as L-beams, and interior beams as T-beams.
- L- and T-beams save weight, and provide a greater lever arm for flexural strength and stiffness.
- Care is required to ensure that the beam and slab are properly held together - hence attention to vertical and horizontal shear force actions is required.

NEXT

14. Member Strength of Columns

(Lecture 15. on ‘Compression rebar for bending’ will be on Monday next week)